2. Data

In our research report, we use Walmart as the representative of offline seller and Amazon as that of online seller. We focus on the daily log return of the Walmart stock and Amazon stock from January 3th, 2010 to December 30th, 2016, with 1761 observations. All the data are pulled from Wind Financial Terminal.

Let be the log return of an asset at time index *t*, and the formula we use to calculate the log return is



where is the close price on day and is the close price on day .

The descriptive statistics and time plots are shown in Table 1 and Figure 1.

From Table 1, we provide the mean log return and its standard deviation for each stock, and Figure 1 shows the time plots of the log returns for Amazon and Walmart. From the plots, the return series for both stocks appear to be stationary and random.

3. Model Establishment

3.1 Model Specification

Figure 2(a) shows the sample ACF of the log returns for Amazon stock, which suggests no significant serial correlations. Figure 2(b), showing the sample PACF of the log returns, also confirms our conclusion of no serial correlation. Then, we plot the sample ACF of the squared log returns for Amazon stock in Figure2(c). Combing the three plots, it seems that the log returns are neither serially correlated nor dependent. Similar results can also be found for Walmart stock in Figure 3 that the log returns for Walmart stock are serially uncorrelated and independent.

3.2 Mean Equation

The observations above suggest that the daily log returns of Amazon stock follow an ARMA(0,0) model, in other words, a white noise series. This is in agreement with the result suggested by the sample ACF in Figure 2(c) that all sample ACFs are close to zero. Therefore, we propose a mean equation that is simply a constant plus innovations,

where is the log return of an asset at time index *t*, is the sample mean of and is the residuals of the mean equation. The squared series is then used to check for conditional heteroscedasticity (ARCH effects). We perform the usual Ljung–Box statistics to the {} series. The null hypothesis is that the first *m* lags of ACF of the series are zero.

The Ljung–Box statistics of the series shows ARCH effects with Q(5) = 9.6519, the value of which is close to zero.

3.3 Volatility Equation

Here we entertain an ARCH(1) model and a GARCH(1,1) model for the volatility and we specify the modelas the following:

ARCH(1):

GARCH(1,1):

Table 2 Ljung-Box tests for ARCH(1) and GARCH(1, 1) models for AMZN Stock

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | Standardized residuals | | | Squared standardized residuals | | |
|  |  | Q(10) | Q(15) | Q(20) | Q(10) | Q(15) | Q(20) |
| ARCH(1) | statistic | 4.7873 | 10.2395 | 12.8693 | 3.9721 | 4.7376 | 5.1054 |
| p.value | 0.9049 | 0.8044 | 0.8829 | 0.9486 | 0.9941 | 0.9997 |
| GARCH(1, 1) | statistic | 4.1734 | 10.2365 | 12.8473 | 3.0443 | 4.2163 | 4.7107 |
| p.value | 0.9392 | 0.8046 | 0.8838 | 0.9804 | 0.99694 | 0.99983 |

Table 3 Results of Estimation of Two Volatility Models for AMZN Stock

|  |  |  |
| --- | --- | --- |
|  | Dependent Variable | |
|  | ARCH(1) | GARCH(1,1) |
|  | 0.1153\*  (0.0464)  3.5151\*\*\*  (0.1491)  0.1680\*\*\*  (0.0364) | 0.1367\*\*  (0.0460)  1.0199\*\*  (0.3435)  0.1286\*\*\*  (0.0315)  0.6373\*\*\*  (0.0984) |
| Log Likelihood | -3724.86 | -3720.01 |
| Akaike Inf. Crit.  Bayesian Inf. Crit. | 4.233793  4.243118 | 4.229426  4.241859 |

Based on the results of log likelihood, AIC, and BIC for ARCH(1) model and GARCH(1,1) model, GARCH(1,1) is slightly more appropriate.

Consequently, we obtain a GARCH(1,1) to model the volatility:

where the standard errors of the parameters are 0.3435, 0.0315, 0.0984, respectively.

In conclusion, we propose the following mean equation and conditional heteroskedasticity model for AMZN stock

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